

MID TERM EXAMINATION, DECEMBER 2021

CLASS : II PUC

SUBJECT : MATHEMATICS (35)

MARKS : 100

TIME : 3 Hrs. 15 Mins.

Instructions :

1. The question paper has five parts namely A, B, C, D and E. Answer all the parts.
2. Use graph sheets for the question on Linear Programming in Part - E.

PART A

I Answer all the Ten questions :

10x1=10

1. Is the relation R on the set {1, 2, 3} given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ symmetric? Justify your answer.
2. Write the range of $\cos^{-1}x$.
3. Define a scalar matrix.
4. Construct a 2×2 matrix whose elements are given by $a_{ij} = \frac{i}{j}$.
5. If A is a square matrix of order 3 then find the value of $|\text{adj } A|$.
6. Differentiate $\sin(ax + b)$ w.r.t. x
7. If $y = \tan \sqrt{x}$ then find $\frac{dy}{dx}$
8. Write the antiderivative of $\frac{1}{x}$; $x \neq 0$ w.r.t x
9. Evaluate : $\int \sec^2(7 - 4x) dx$.
10. Define objective function in Linear Programming problem.

PART B

II Answer any TEN of the following questions :

10x2=20

11. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ on set of natural numbers given by $f(1) = f(2) = 1$ and $f(x) = x - 1$; for every $x > 2$ is onto but not one-one.
12. Evaluate : $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$
13. Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$
14. Evaluate : $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$
15. Find the values of x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$
16. Find the area of the triangle with vertices (1, 0), (6, 0) & (4, 3) using determinants.
17. Find $\frac{dy}{dx}$ if $ax + by^2 = \cos y$
18. Find the second order derivative of the function $\log x$ w.r.t. x
19. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(P.T.O)

20. Find $\frac{dy}{dx}$ if $xy = e^{x-y}$

21. Find the equation of the tangent to the curve $y = x^3$ at $(1, 1)$

22. Evaluate : $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

23. Evaluate : $\int \frac{1}{x + x \log x} dx$

24. Evaluate : $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$

PART C

III Answer any TEN of the following questions :

10x3=30

25. Show that the relation R on the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

26. Check whether the relation R on the set of real numbers defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

27. Express $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

28. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ then find $(A + 2B)'$.

29. If $x = 2at^2$; $y = at^4$ find $\frac{dy}{dx}$

30. If $y = x^{\sin x}$; $x > 0$ find $\frac{dy}{dx}$

31. Prove that if a function is differentiable at a point 'c' then it is also continuous at that point.

32. If $x = a \cos^3 \theta$; $y = a \sin^3 \theta$ then prove that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$

33. Find the interval in which the function f given by $f(x) = x^2 - 4x + 6$ is
(a) strictly increasing.
(b) strictly decreasing.

34. Find two positive numbers whose sum is 15 and sum of whose squares is minimum.

35. Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$

36. Evaluate $\int \sin^3 x dx$

37. Evaluate : $\int \frac{x dx}{(x+1)(x+2)}$

38. Evaluate : $\int x \sin x dx$

(P.T.O)

PART D

IV Answer any SIX of the following questions :

6x5=30

39. Check whether the function $f: \mathbb{R} \rightarrow \mathbb{R}$ on set of real numbers defined by $f(x) = 3 - 4x$ is one-one, onto or objective. Justify your answer.

40. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ calculate $(A + B)C$,

AC and BC . Also verify $(A + B)C = AC + BC$.

41. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

then compute $(A + B)$ and $(B - C)$. Also verify $A + (B - C) = (A + B) - C$

42. Solve the following system of linear equations by matrix method :

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

43. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ then find A^{-1} . Using A^{-1} , solve the system of equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

44. If $y = 3e^{2x} + 2e^{3x}$ show that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

45. If $y = (\tan^{-1} x)^2$ show that $(x^2+1)^2y_2 + 2x(x^2+1)y_1 = 2$

46. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

47. Find the integral of $\frac{1}{x^2+a^2}$ w.r.t x and evaluate $\int \frac{dx}{x^2+2x+2}$.

48. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ w.r.t. x and hence find $\int \frac{dx}{\sqrt{9-25x^2}}$

(P.T.⊙)

PART E

V Answer any ONE of the following questions :

1x10=10

49. a) Solve the following linear programming problem graphically :

$$\text{Maximize : } Z = 4x + y$$

$$\text{Subject to constraints : } x + y \leq 50$$

$$3x + y \leq 90$$

$$x, y \geq 0$$

(6)

b) Find the value of 'k' if $(fx) = \begin{cases} kx^2 ; x \leq 2 \\ 3 ; x > 2 \end{cases}$ is continuous at $x = 2$. **(4)**

50. a) Show that following problem graphically :

$$\text{Minimis : } Z = -3x + 4y$$

$$\text{Subject to constraints : } x + 2y \leq 8$$

$$3x + 2y \leq 12$$

$$x, y \geq 0$$

(6)

b) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies $A^2 - 5A + 7I = 0$ then find A^{-1} using this equation ; where 'I' is identity matrix and 'O' is zero matrix of order 2 respectively. **(4)**

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