I PUC MID-TERM EXAMINATION, OCTOBER-2023 (SET-2)

Time : 3 Hrs. 15 Mins. **SUBJECT : MATHEMATICS (35)** **Max Marks : 80**

Instructions :

1) **The question paper has five parts namely A, B, C, D and E. Answer all the parts.**
2) **Part A has 15 Multiple choice questions. 5 Fill in the blank questions.**

PART - A

I Answer all the multiple choice questions : **15x1=15**

1) The roster form of the set $C = \{x : x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\}$ is

A) {0, 1, 2, 3} B) {0, 1, 2, 3, 4} C) {1, 2, 3, 4} D) None of these

2) Choose the correct statement

A) $\{a, b\} \not\subset \{b, c, a\}$ B) $\{a\} \in \{a, b, c\}$ C) $a \in \{a, b, c\}$ D) $\{c, \phi\} \subset \{a, b, c\}$

3) Let U be the universal set, A be the subset of U and ϕ the null set, then $\phi' \cap A =$

A) A B) ϕ C) U D) A'

4) Let A = {x, y, z} and B = {1, 2}, then the number of relations from A to B is

A) 10 B) 32 C) 64 D) 16

5) The range of the function $f(x) = x^2 + 2$ (x is a real number) is

A) $(-\infty, \infty)$ B) $(-\infty, 2]$ C) $(-2, 2]$ D) $[2, \infty)$

6) The approximate value of 1 radian is

A) 58° B) 60° C) 57° 16' D) 57° 46'

7) If $\cot x = \frac{-5}{12}$, x lies in the third quadrant, then sec x is

A) $-13/5$ B) $17/5$ C) $5/13$ D) $5/12$

8) The value of $(1-i)-(-1+i6)$ is

A) $2-7i$ B) $2+7i$ C) $-2-7i$ D) $-2+7i$

9) The multiplicative inverse of $4-3i$ is

A) $\frac{4+3i}{25}$ B) $\frac{-4+3i}{25}$ C) $\frac{-4-3i}{25}$ D) $\frac{-4+3i}{25}$

10) If x is an integer then the solution set of the inequality $5x-3 < 7$ is

A) {0, 1, −1, −2, −3, ...} B) {−1, −2, −3, ...}
C) {0, 1, 2, 3, 4, ...} D) {0, −1, −2, −3, ...}

11) For $x \in R$, solution set for the inequality $3(2-x) \geq 2(1-x)$ is

A) $(4, \infty)$ B) $(-\infty, 4]$ C) $(\infty, 4]$ D) $[4, \infty)$

12) The slope of the line $\frac{x}{3} + \frac{y}{2} = 1$ is

A) 3 B) 2 C) $-2/3$ D) $3/2$

13) The slope of the line making an inclination of 60° with the positive direction of x-axis is

A) $\sqrt{3}$ B) $1/\sqrt{3}$ C) 1 D) 0

(P.T.O.)

14) The radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$ is
A) 8 B) 5 C) 6 D) 7

15) The coordinates of the focus of the parabola $x^2 = 8y$ is
A) (8, 0) B) (0, 8) C) (0, 2) D) (2, 0)

II Fill in the blanks by choosing the appropriate answer from those given in the brackets: **5x1=5**

$\left(\frac{2}{5},\ -11,\ 0,\ \frac{-3}{2},\ 300^\circ\right)$

16) A function is defined by $f(x) = 2x - 5$ then the value of $f(-3)$ is _____.

17) The degree measure corresponding to $\frac{5\pi}{3}$ radian is _____.

18) The value of $\cos(-1710^\circ)$ is _____.

19) The slope of the line joining the points $(3, -2)$ and $(-1, 4)$ is _____.

20) The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$ is _____.

PART-B

III Answer any SIX questions : **6x2=12**

21) Write the following intervals in set builder form
(i) $[-7, 8]$ (ii) $(-3, 5)$

22) Draw the Venn diagram for $A' \cup B'$.

23) Show that the statement "If $A \subset B$ and $B \in C$, then $A \in C$" is false by giving an example.

24) The minute hand of a watch is 1.5 cm long. How far does the tip move in 40 minutes ? $(\text{Use } \pi = 3.14)$.

25) Prove that $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$.

26) Express $(5 - 3i)^3$ in the form a + ib.

27) If $x - iy = \sqrt{\frac{a - ib}{c - id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$.

28) Solve the inequality $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$ and represent the graph of the solution on number line.

29) The marks obtained by a student of class XI in first and second terminal examination are 62 and 48, respectively. Find the minimum marks he should get in the annual examination to have an average of at least 60 marks.

30) Find the equation of a line for which $\tan\theta = \frac{1}{2}$ where θ is the inclination of the line and x intercept is 4.

31) Reduce the equation $3x + 2y - 12 = 0$ into intercept form and also find their intercepts on the axes.

PART -C

IV Answer any SIX questions : **6x3=18**

32) Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$, then prove that $(A \cup B)' = A' \cap B'$.

33) Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Find $V - B$, $B - V$ and $B \cup V$.

34) Write the relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in the roster form. Also find the domain and range of the relation R.

(P.T.O.)

35) Let f, g : R → R be defined, respectively by f(x) = x + 1, g(x) = 2x − 3. Find (f + g) (x), (f − g) (x) and $\left(\frac{f}{g}\right)(x)$.

36) Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$.

37) Show that $\cos 3x = 4\cos^3 x - 3\cos x$.

38) Find the conjugate of complex number $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ in the form a + ib.

39) Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is less than 11.

40) The angle between the lines is $\pi/4$ and slope of one of the lines is $1/2$. Find the slope of the other line.

41) Find the equation of the line which makes intercepts a and b on the x and y axes respectively.

42) Find the equation of the parabola with vertex (0, 0) passing through the point (2, −3) and symmetric with respect to y-axis.

PART-D

V Answer any FOUR questions : **4x5=20**

43) Define the greatest integer function. Draw the graph of the function. Write the domain and range of the function.

44) Define the identify function. Draw the graph of the function. Write the domain and the range of the function.

45) Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$.

46) Prove that $(\cos x - \cos)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$.

47) Derive the formula for the length of perpendicular drawn from any point (x_1, y_1) to the line ax + by + c = 0.

48) Find the angle between the lines having slopes m_1 and m_2 and hence using the formula find the angle between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

49) Find the equation of the circle passing through the points (2, −2) and (3, 4) and whose centre is on the line x + y = 2.

50) Find the coordinates of the foci, eccentricity, length of the axes and length of latus rectum of the ellipse $4x^2 + 9y^2 = 36$.

PART-E

VI Answer the following questions :

51) Prove geometrically that cos (x + y) = cos x cos y − sin x sin y and hence find the value of cos (x − y). **(6)**

OR

Define hyperbola and derive the equation of hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

52) Prove that $\frac{\tan\left(\pi/4 + x\right)}{\tan\left(\pi/4 - x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$. **(4)**

OR

The slope of the line is double of slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines.

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