Class: I PUC	Mid Term Exam Dec-2024 Subject: Mathematics (35)	Max. time: 3hr Max. Marks: 80

PART - A

I. **Answer all the multiple choice questions:** (15 x 1 = 15)

1. If A and B are two sets, then $A \cap (A \cup B)'$ equals
   a) A b) B c) $\emptyset$ d) $A \cap B$
2. Let $n(A) = 3, n(B) = 2$, then the number of relations from A to B is
   a) 64 b) 8 c) 9 d) 32
3. If A lies in the second quadrant and $3\tan A + 4 = 0$, then the value of $(2\cot A - 5\cos A + \sin A) =$ _____
   a) $\frac{-53}{10}$ b) $\frac{37}{10}$ c) $\frac{23}{10}$ d) $\frac{7}{10}$
4. The value of $i^{57} + \frac{1}{i^{125}}$ is,
   a) 0 b) $2i$ c) $-2i$ d) 2
5. The solution of $x + \frac{x}{2} + \frac{x}{3} < 11$ is
   a) $(-\infty, 6)$ b) $(-\infty, 6]$ c) $(6, \infty)$ d) $[6, \infty)$
6. For any two positive numbers, we have
   a) AM ≤ GM b) AM ≥ GM c) AM = ¾ GM d) None of these
7. The point $(-3, 1, 2)$ lies in the octant
   a) I b) II c) III d) IV
8. Events A and B are said to be mutually exclusive if:
   a) $P(A \cup B) = P(A) + P(B)$ c) $P(A \cap B) = P(A) + P(B)$
   b) $P(A \cup B) = 0$ d) None of these
9. The letters of the word "SOCIETY" are placed at random in a row. The probability that three vowels occur together is
   a) $\frac{1}{7}$ b) $\frac{2}{7}$ c) $\frac{3}{7}$ d) $\frac{4}{7}$
10. Equation of the line passing through the point (1, 2) and perpendicular to the line $x + y + 1 = 0$ is
    a) $y - x + 1 = 0$ c) $y - x - 1 = 0$
    b) $y - x + 2 = 0$ d) $y - x - 2 = 0$
11. If the focus of a parabola is $(0, -3)$ and its directrix is $y = 3$, then its equation is
    a) $x^2 = -12y$ b) $x^2 = 12y$ c) $y^2 = -12x$ d) $y^2 = 12x$
12. The standard deviation of first 10 natural numbers is:
    a) 5.5 b) 3.87 c) 2.97 d) 2.87
13. The third term of a GP is 4. The product of first five terms is
    a) 4^3 b) 4^5 c) 4^4 d) None of these

In question no. 14 and 15, a statement of assertion is followed by a statement of reason is given. Choose the correct answer out of the following choices:

(a) Assertion and reason both are correct, and reason is the correct explanation of assertion.

(b) Assertion and reason both are correct, but reason is not the correct explanation of assertion.

(c) Assertion is correct statement, but reason is wrong.

(d) Assertion is wrong statement, but reason is correct.

14. **Assertion:** Coordinates $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

Reason: Opposite sides of a parallelogram are equal and diagonals are not equal.

15. **Assertion:** $\lim_{x\to 0} \frac{\sin 5x}{\sin x} = 5$

Reason: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

II. **Fill in the Blanks** (5 x 1 = 5)

$$\left\{0, \frac{\sqrt{3}}{2}, \sqrt{3}, 1, \frac{1}{\sqrt{3}}, \frac{4}{\sqrt{3}}\right\}$$

16. If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ then the value of y is_____.

17. The value of $sin^2 \frac{5\pi}{12} - sin^2 \frac{\pi}{12}$ is_____.

18. The slope of line making inclination 60° with the positive direction of x-axis is____.

19. The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is_____.

20. The derivative of $x^2 - 4x + 4$ at $x = 2$ is_____.

PART - B

III. Answer any six questions (2 x 6 = 12)

21. If $A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 4, 5, 6, 8\}$. Find $A - B$ and $B - A$.

22. Let $A = \{1, 2\}, B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have?

23. Prove that $sin^2 \frac{\pi}{6} + cos^2 \frac{\pi}{3} - tan^2 \frac{\pi}{4} = -\frac{1}{2}$

24. Express $(1 - i)^4$ in $a + ib$ form.

25. Solve the inequality $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$ and show the graph of solution on number line

26. How many 3-digit even numbers can be formed from the digits $1, 2, 3, 4, 6, 7$ if no digit is repeated?

27. Expand $\left(x^2 + \frac{3}{x}\right)^4$ using Binomial Theorem.

28. The sum of first three terms of a G.P. is 16 and the sum of next three terms is 128. Find the sum of n terms of the GP

29. Find the equation of the line parallel to the line $3x - 4y - 2 = 0$ and passing through the point (-2, 3)

30. Evaluate $\lim_{x\to 1}\left[\frac{x^{15}-1}{x^{10}-1}\right]$
31. Events E and F are such that $P(not\ E\ or\ not\ F) = 0.25$. State whether E and F are mutually exclusive.

PART - C

IV. **Answer any six questions:** (6 x 3 = 18)

32. If $U = \{1,2,3,4,5,6,7,8,9\}\ A = \{2,4,6,8\}\ and\ B = \{2,3,5,7\}$ Verify that $(A \cup B)' = A' \cap B'$
33. Find the domain and range of the following real function $f(x) = \sqrt{9-x^2}$
34. Prove that $\sin 3x = 3\sin x - 4\ sin^3 x$
35. Express the following in the form of $a + ib$: $\frac{5+i\sqrt{2}}{1-i\sqrt{2}}$.
36. If $\tan x = -\frac{5}{12}, x$ lies in II quadrant, find the values of other five trigonometric functions
37. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.
38. If A.M. and G.M. of two positive numbers 'a' and 'b' are 34 and 6 respectively. Find the numbers.
39. The vertices of a triangle are A (10, 4), B (- 4, 9), and C (-2, -1). Find the equation of the altitude through A
40. Find the coordinates of the focus, the equation of the directrix and latus rectum of the parabola $y^2 = -8x$
41. Show that the points $P(-2,3,5), Q(1,2,3)\ and\ R(7,0,-1)$ are collinear.
42. Find the derivative of $y = \sin x$ with respect to x from first principle method.

PART - D

V. **Answer any four questions:** (4 x 5 = 20)

43. Define Modulus function. Draw the graph of it. Also write its domain and Range
44. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$
45. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of
    i) Exactly 3 girls
    ii) At least 3 girls
    iii) At most 3 girls
46. State and prove 'Binomial Theorem' for positive integral index 'n'
47. If $p\ and\ q$ are the lengths of perpendiculars from the origin to the line $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\ cosec\ \theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

48. Prove Geometrically that

$\lim_{x \to 0} \frac{\sin x}{x} = 1, where\ x$ being measured in radians.

49. Calculate the mean deviation about the mean for the following data

Marks Obtained	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**
Number of students	2	3	8	14	6	3	2

50. A bag contains 9 discs of which 4 are Red, 3 are Blue and 2 are Yellow. The discs are similar in shape and size. A disc is thrown at random from the bag. Calculate the probability that it will be:
    a) Red
    b) Yellow
    c) Blue
    d) Not Blue
    e) Either Red or Blue

PART - E

VI. **Answer the following questions:**

51. Prove geometrically that

$\cos(x + y) = \cos x \,.\, \cos y - \sin x . \sin y$

OR

Derive the equation of ellipse in the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (6M)

52. Find the sum to 'n' terms of the sequence 8, 88, 888, ….

OR

Find the derivative of $\frac{\cos x}{1+\sin x}$ w.r.t x. (4M)

**********ALL THE BEST**********