

DISTRICT LEVEL I PUC ANNUAL EXAMINATION FEB - 2020

Time: 3 Hrs. 15 Mins.

Sub: MATHEMATICS (35)

Max. Marks: 100

General Instructions:

1. The question paper has Five parts, namely A, B, C, D and E. Answer all the parts.
2. Use the graph sheet for the question on linear inequalities in Part - D.

Part - A**I. Answer ALL the following questions:****10 × 1 = 10**

1. Write the set $\{x : x \in R, -4 < x \leq 6\}$ as interval.
2. If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ find the value of x .
3. If $\cot x = \frac{3}{4}$ and x lies in the third quadrant, find $\sin x$.
4. Find the multiplicative inverse of $2 - 3i$.
5. Find 'n' if ${}^nC_8 = {}^nC_2$.
6. Find the fifth term of the sequence defined by $a_n = \frac{2n-3}{6}$.
7. Find the slope of line passing through the points, (3, 2) and (-1, 4).
8. Evaluate $\lim_{x \rightarrow 0} \left[\frac{e^{4x} - 1}{x} \right]$.
9. Identify the quantifier in the statement, "There exist a number which is equal to its square".
10. Define sample space of a random experiment.

Part - B**II. Answer any TEN of the following questions:****10 × 2 = 20**

11. Taking the set of natural number as universal set, write down the complement of the set $\{x : x + 5 = 8\}$.
12. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$ and $C = \{11, 13, 15\}$ find $A \cap (B \cup C)$.
13. Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have?
14. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm
 $\left(\text{use } \pi = \frac{22}{7} \right)$.
15. Prove that $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = \frac{-1}{2}$.

16. Solve $2x^2 + x + 1 = 0$.
17. Solve $3x - 2 < 2x + 1$. Show the graph of the solution on number line.
18. Find the equation of the line parallel to the line $3x - 4y - 2 = 0$ and passing through the point $(-2, 3)$.
19. Find the angle between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.
20. Show that the points $(-2, 3, 5)$ $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.
21. Evaluate $\lim_{x \rightarrow 1} \left[\frac{x^{15} - 1}{x^{10} - 1} \right]$.
22. Write the converse and contrapositive of the statement, "If a number 'n' is even then n^2 is even".
23. If the co-efficient of variation of the distribution is 60 and it's standard deviation is 21. Find the arithmetic mean of the distribution.
24. Events E and F are such that $P(\text{not } E \text{ or not } F) = 0.25$. State whether E and F are mutually exclusive.

Part - C

III. Answer any TEN of the following questions:

10 × 3 = 30

25. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1 , 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2 . Find the number of individuals exposed to
 - (i) Chemical C_1 or Chemical C_2 .
 - (ii) Chemical C_1 but not Chemical C_2
26. Let $f, g : R \rightarrow R$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$.
Find (i) $f + g$ (ii) $f - g$ (iii) $\frac{f}{g}$
27. Find the general solution of the equation $2\cos^2 x + 3\sin x = 0$.
28. If $x - iy = \sqrt{\frac{a - ib}{c - id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$.
29. Convert the complex number $Z = -1 + i$ in polar form.
30. Find the number of different 8 - letters arrangements that can be made from the letters of the word "DAUGHTER" so that
 - (i) All vowels occur together
 - (ii) All vowels do not occur together.
31. Find the term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x} \right)^6$.
32. Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.
33. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio.
34. Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$.

35. Find the derivative of ' $\sin x$ ' with respect to x , from first principles.
36. Verify by the method of contradiction that " $\sqrt{7}$ is irrational".
37. A committee of two persons is selected from two men and two women. What is the probability that the committee will have (a) no men (b) one man (c) two men.
38. A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine (i) $P(\text{not } A)$ (ii) $P(\text{not } B)$ (iii) $P(A \text{ or } B)$.

Part – D

IV. Answer any SIX of the following questions:

6 × 5 = 30

39. Define signum function. Draw the graph of it and write down its domain and range.
40. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$.
41. Prove by mathematical induction $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}, \forall n \geq N$
42. Solve the inequalities graphically, $5x + 4y \leq 20, x \geq 1, y \geq 2$.
43. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of (i) Exactly 3 girls (ii) At least 3 girls? (iii) atmost 3 girls?
44. State and prove Binomial theorem for positive integral index.
45. Derive the formula of distance of a point $P(x_1, y_1)$ from the line $Ax + By + C = 0$.
46. Derive the formula to find the coordinates of a point that divides the line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ internally in the ratio $m : n$.
47. Prove geometrically that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, where x is measured in radians.
48. Calculate the mean deviation about the mean for the following data:
- | | | | | | | | |
|--------------------|-------|-------|-------|-------|-------|-------|-------|
| Marks obtained | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Number of students | 2 | 3 | 8 | 14 | 8 | 3 | 2 |

Part – E

V. Answer any ONE of the following questions:

1 × 10 = 10

49. (a) Prove Geometrically that $\cos(x + y) = \cos x \cos y - \sin x \sin y$. Hence show that $\cos 2x = \cos^2 x - \sin^2 x$ [6M]
- (b) Find the sum to 'n' terms of the series $3 \times 8 + 6 \times 11 + 9 \times 14 + \dots$ [4M]
50. (a) Define ellipse as a set of points. Derive its equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [6M]
- (b) Find the derivative of $\frac{\cos x}{1 + \sin x}$ w.r.t. x . [4M]
